Finding the Complex Permittivity as a Function of Energy Using Angular Straggling

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Team Scattered Stragglers from *Argentina, Canada, India, Malaysia, Nepal*

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1 Introduction

Straggling (Landau) functions describe the probability distribution of energy loss of charged particles traversing matter and are widely used for particle identification in the relativistic velocity region [1]. The differential cross section $d\sigma/dE$ characterizes the energy transfer in individual collisions and is often described by the Fermi–Virtual Photon (FVP) model. We measure the angular spread of particles after traversing a target and infer the underlying cross section using special relativity and numerical.

Objectives

1. Test the FVP model. Compare experimental cross-sections to the FVP prediction by checking the known complex permittivity ϵ of the medium.

2. Measurement of the complex permittivity. The complex dielectric constant $\epsilon(\Delta E)$ of tungsten is extracted as a function of deposited energy. A scattering-based measurement may extend the accessible energy range.

2 Motivation

We are a group of physics enthusiasts scattered around the world driven by our shared passion of discovering the physics of our world. We want to use angular straggling and Tungsten's differential cross-section to determine its complex permittivity. Tungsten is highly rewarding practicality-wise, used in nuclear fusion reactors and sitting inside nearly every semiconductor chip as a critical interconnect metal.

Our participation at CERN would allow us to move beyond theoretical study and experience how research is actually conducted.

3 Theoretical Background

The FVP model [2] expresses the single-collision DCS in terms of the complex dielectric constant $\epsilon = \epsilon_1 + i\epsilon_2$:

$$\begin{aligned}\frac{d\sigma}{dE} = \frac{\alpha}{\beta^2\pi}\Bigg[&\frac{\sigma_\gamma(E)}{EZ}\ln\Big(((1-\beta^2\epsilon_1)^2+\beta^4\epsilon_2^2)^{-1/2}\Big) + \frac{1}{N\hbar c}\left(\beta^2 - \frac{\epsilon_1}{|\epsilon|^2}\right)\Theta \\ &+\frac{\sigma_\gamma(E)}{EZ}\ln\left(\frac{2mc^2\beta^2}{E}\right) + \frac{1}{E^2}\int_0^E \frac{\sigma_\gamma(E')}{Z}\,dE'\Bigg]\end{aligned} \tag{1}$$

$$\sigma_\gamma(E) = \frac{ZE\epsilon_2}{\hbar Nc\sqrt{\epsilon_1}}, \qquad \Theta = \arg(1-\epsilon_1\beta^2 + i\epsilon_2\beta^2) \tag{2}$$

Since we measure angular deflections, we use the Jacobian

$$\frac{d\sigma}{d\theta} = \frac{d\sigma}{dE}\left|\frac{dE}{d\theta}\right|, \tag{3}$$

where $dE/d\theta$ follows from relativistic two-body kinematics [3] (see Appendix B). Normalising gives the single-collision PDF $P_1(\theta) = \sigma_{\text{tot}}^{-1}\,d\sigma/d\theta$.

Projecting onto one transverse axis via the Abel transform yields $g_1(\Theta_x)$, the 1D single-scatter distribution:

$$g_1(\Theta_x) = 2\int_{|\Theta_x|}^{\infty} \frac{P_1(\theta)\,\theta\,d\theta}{\sqrt{\theta^2-\Theta_x^2}}. \tag{4}$$

After n collisions this becomes $g_n = g_1^{\otimes n}$. Since collisions in thickness x are Poisson-distributed with mean $m_c = xN_{\text{atom}}\sigma_{\text{tot}}$, the straggling distribution is

$$P_{\text{strag}}(\Theta_x) = \sum_{n=1}^{\infty}\frac{m_c^n e^{-m_c}}{n!}\,g_n(\Theta_x). \tag{5}$$

Convolution with the beam profile $n(x_0)$ gives the measured distribution $R(\theta)$.

Inversion proceeds in three steps: (1) Wiener deconvolution of $R(\theta)$ recovers P_{strag}; (2) a candidate $d\sigma/d\theta$ is optimised by minimising the residual between the forward-modelled $P_{\text{strag}}^{\text{pred}}$ and the data, initialised from the Rutherford cross section; (3) the Jacobian

$$\frac{d\sigma}{dE} = \frac{d\sigma}{d\theta}\left|\frac{d\theta}{dE}\right| \tag{6}$$

yields the energy-loss DCS, from which ε is extracted by inverting equation (1).

4 The Experiment

4.1 Overview

The experiment measures how much protons deflect when fired into thin Tungsten slabs. The beam travels along the z-axis, with the Tungsten plate sitting at $z = 0$ mm. Upstream means negative-z (before the plate), downstream means positive z (after).

4.2 Experimental Design and Setup

4.2.1 Upstream Section - Before the Target

1. **S1 - Scintillator Trigger** ($z = -1500\,\text{mm}$)
   Used to detect the arrival of particles.

2. **C1 / C2 - First and Second Cherenkov Detectors**
   ($z = -1300\,\text{mm}$, $z = -900\,\text{mm}$)
   C1 is a cherenkov detector at reduced pressure, while C2 is set to a higher pressure to identify protons passing through the plate.

3. **DWC1 / DWC2 - First and Second Delay Wire Chambers**
   ($z = -330\,\text{mm}$, $z = -300\,\text{mm}$)
   Measures the first and second point of the incoming track to determine the incoming direction vector of the protons, $\hat{n}_{\text{in}}$.

4.2.2 The Target - At $z = 0$ mm

The target is a thin Tungsten plate (W), with dimensions of $3\,cm \times 3\,cm$. The thickness is varied between runs, from 1 mm to 10 mm in increments of 1 mm.

4.2.3 Downstream Section - After the Target

1. **DWC3 / DWC4 - Third and Fourth Delay Wire Chambers**
   ($z = 200\,\text{mm}$, $z = 250\,\text{mm}$)
   With this pair, the outgoing direction vector of the proton, $\hat{n}_{\text{out}}$ can be determined.

   The scattering angle is then the angle between the two track vectors.

$$\Theta = \arccos(\hat{n}_{\text{in}} \cdot \hat{n}_{\text{out}})$$

2. **S2 - Scintillator Trigger** ($z = 600\,\text{mm}$)
   The detection of protons by S1 and S2 is utilised as a trigger condition for the DAQ to read out all detectors, given that they coincide within a 100 ns window.

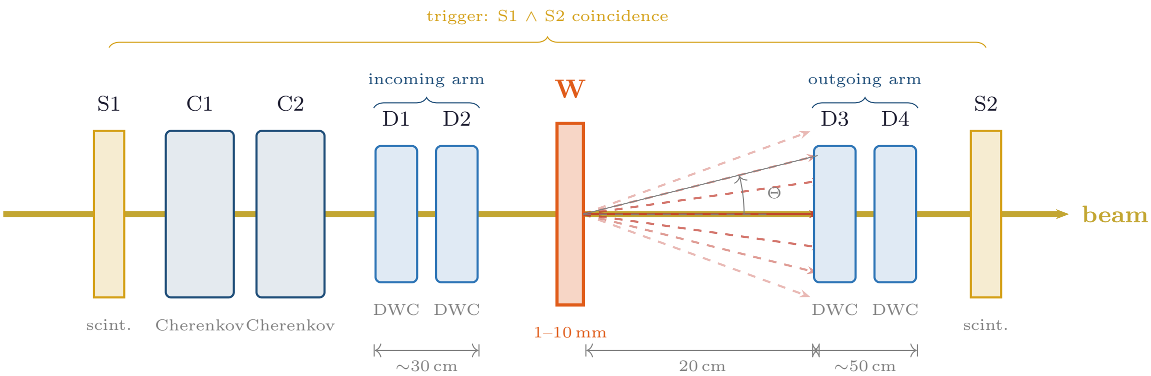

Figure 1: Beamline layout. Protons travel left to right. Cherenkov detectors C1, C2 select protons by PID (clean at 4 GeV/c: $\Delta\beta_{\pi p} \approx 0.027$). DWC pairs D1/D2 and D3/D4 reconstruct the incoming and outgoing tracks respectively; D3/D4 are placed 200 mm downstream to achieve >99 % geometric acceptance at all Tungsten thicknesses. The deflection angle Θ is measured per event. The Tungsten foil (W) is interchangeable across ten thicknesses.

5 Data Analysis

From reconstructed DWC hit positions, the scattering angle is obtained as $\theta = \tan^{-1}(r/20\,\mathrm{cm})$ and binned to produce $dN/d\Omega$. Beam divergence is removed by deconvolving the distribution with a Gaussian beam profile ($\sigma_{\mathrm{ang}} = 0.25\,\mathrm{mrad}$), yielding the angular straggling function $P(\theta)$.

The single-scattering cross section $d\sigma/d\Omega$ is recovered by fitting a Poisson-weighted forward model to $P(\theta)$, where the predicted straggling is the Poisson-weighted sum of n-fold two-dimensional self-convolutions of a candidate distribution. The normalization is fixed using the relativistic Coulomb cross section at the largest measured angle. The energy-loss cross section $d\sigma/dE$ is then obtained via the Jacobian transformation (6), and the complex permittivity ε is extracted by numerically solving the Allison–Cobb FVP equation (1).

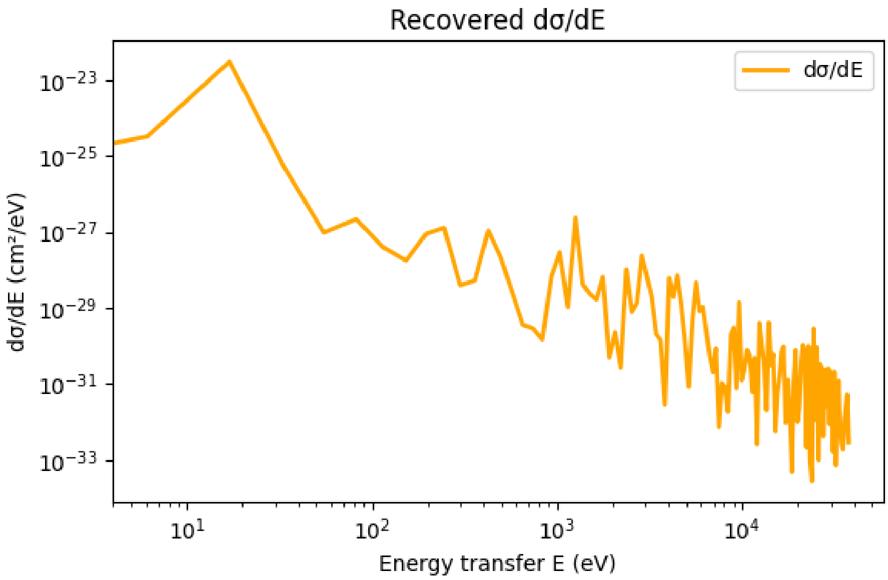

Figure 2: The best differential cross section obtained from our current numerical simulations. Due to limited simulation statistics, the reconstructed cross section is poorly constrained at very small angles. A higher particle flux at CERN will allow us to obtain significantly improved statistics in this region.

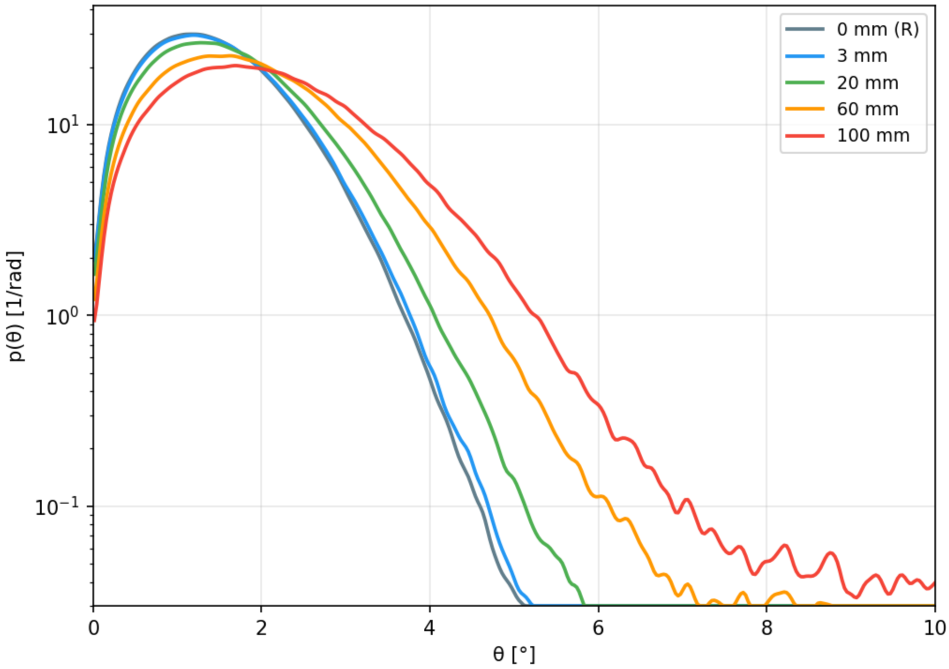

Figure 3: Normalised angular distributions of 4 GeV/c protons transmitted through tungsten plates of varying thickness (0–100 mm), measured at DWC3 ($z = 200$ mm downstream of target). The 0 mm run represents the instrument response $R(\theta)$ comprising beam divergence, air scatter, and detector material contributions.

6 Schedule

Day	Target	Activity
1 AM	—	Detector alignment; DWC timing calibration.
1 PM	None	Baseline beam divergence $B(\theta)$; Cherenkov threshold tuning.
2	W 6, 10 cm	Angular straggling data at 4 GeV/c for both tungsten thicknesses.
3	W 6 cm	Momentum scan: $p = 1, 3, 6, 10$ GeV/c; probes β-dependence of FVP cross-section.
4	Pb 6, 10 cm	Angular straggling data for both lead thicknesses; offline processing of tungsten runs.
5 AM	Pb 6 cm	Momentum scan: $p = 1, 3, 6, 10$ GeV/c; same β-dependence check for lead.
5 PM	—	Offline analysis of lead data; final model comparison and conclusions.

Table 1: Planned on-site run schedule. Tungsten and lead are each measured at two thicknesses to sample the Landau and Gaussian straggling regimes, and at four momenta to test the β^{-2} dependence predicted by the FVP model. Runs proceed thin-to-thick to detect beam degradation early.

7 What We Hope To Take Away From The Experience

Our group comes from a background of physics and astrophysics Olympiads, some members with past BL4S experience, sparking our desire to pursue physics deeply. While we have already had an enriching experience studying the theory behind this experiment and running simulations, we are eager to conduct our experiment to gain insightful knowledge and exposure to a professional research setting, and learn the hands-on process of running this equipment. We truly believe that firsthand participation at CERN is invaluable for helping us understand what it truly means to work in the field and grow as young aspiring physicists.

8 Acknowledgments

We are grateful for the assistance and guidance of our coaches, Tristan Yan-Klassen and Yash Mehta.

We used LLMs to help fix our first (flawed) attempt at a numerical analysis code because it took many discarded ideas to reach our current proposal, and we did not feel it was possible to take on such a huge undertaking with the limited time available. It took the language models a lot of guidance to reach some decent results, so we would like to learn how to implement a functional method on our own and in a cleaner way.

References

[1] H. Bichsel, "A method to improve tracking and particle identification in TPCs and silicon detectors", Nuclear Instruments and Methods in Physics Research Section A **562**, Received 9 March 2006; accepted 13 March 2006, 154–197 (2006).

[2] W. W. M. Allison and J. H. Cobb, "Relativistic charged particle identification by energy loss", Annual Review of Nuclear and Particle Science **30**, 253–298 (1980).

[3] F. Salvat and J. M. Quesada, "Collisions of nucleons with atoms: calculated cross sections and Monte Carlo simulation", Frontiers in Physics, Published 13 December 2021 (2021).